

Research Article

New Exact Solutions of Nonlinear (3 + 1)-Dimensional Boiti-Leon-Manna-Pempinelli Equation

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Based on Hirota's bilinear structure, we evolute a new protuberance type arrangement of the (3+1)-dimensional Boiti-Boiti-Leon-Manna-Pempinelli equation, which depicts nonlinear wave spreads in incompressible fluid. New lump arrangement is built by applying the bilinear strategy and picking appropriate polynomial. Under various parameter settings, this lump arrangement has three sorts of numerous irregularity waves, blended arrangements including lump waves and solitons are additionally developed. Association practices are seen between lump soliton and soliton. Research demonstrates that soliton can somewhat swallow or release lump waves. The shape and highlights for these subsequent arrangements are portrayed by exploiting the three-dimensional plots and comparing shape plots by picking suitable parameters. The physical significance of these charts is given.

1. Introduction

Numerous analysts in the ongoing years considered numerous kinds of advancement equations portraying distinctive cases in liquid and plasma fields. A wide range of techniques are utilized to examine development equations in (3+1) measurements, for example, Hirota's strategy [1, 2] exponential function method [3, 4], tanh-coth methods and sine-cosine [5, 6], and numerous different techniques. One of the notable equations is the (3+1)-dimensional Boiti-Leon-Manna-Pempinelli (BLMP) equation which depicts the liquid engendering and can be considered as a model for incompressible liquid [7–12]. This equation composes as;

$$v_{yt} + v_{zt} + v_{xxxy} + v_{xxxz} - 3v_x \left(v_{xy} + v_{xz} \right) - 3v_{xx} \left(v_y + v_z \right) = 0$$
(1)

This equation presented by Darvishi et al. [7] as an augmentation of the (2+1)-dimensional equation depicts the (2+1)dimensional connection of the Riemann wave along the yaxis with a long wave proliferated along the x-axis. BLMP equation concentrated by alternate points of view, for example, the utilization of the exponential function method [7, 8]. The precise three-wave arrangements and various soliton arrangements were found by applying the Hirota bilinear technique [9–12]. Lump waves, as exceptional nonlinear wave wonders, have been seen in numerous fields [13–18]. It is huge to most likely find and anticipate protuberance-like waves in applications [19-21]. As of late, exploration on the lump arrangements has pulled in an ever-increasing number of considerations [22-29]. Therefore, theoretically, researches on lump waves are helpful to better understand and predict possible extremes for nonlinear evolution systems. Bilinear technique is a powerful representative calculation to develop

numerous solitons. Very as of late, Ma et al. stretched out this technique to look for lump arrangements, where its basic thought is to pick appropriate polynomial capacities in bilinear frames [30–38]. We are roused to investigate new dynamical properties through examining irregularity answer for nonlinear frameworks. Here, we examine some of irregularity soliton arrangements, their elements, and the vulnerability of communication with different sorts of arrangements utilizing Hirota strategy for (1). By utilizing the complex strategy with two-term truncated arrangement, gathering the coefficients of the eigenfunction and comparing them to zero, we infer the equivalent ansatz in [39, 40]:

$$v(x, y, t, z) = -2\left(\ln\left(\varsigma\left(x, y, t, z\right)\right)_{x}\right)$$
(2)

That is called Cole-Hopf change, where ς is an assistant or test work that will be assumed later. Beginning by substituting from (2) in (1),

$$4\varsigma_{x}\varsigma_{y}\varsigma_{t} + 4\varsigma_{x}\varsigma_{z}\varsigma_{t} + 4\varsigma_{xxx}\varsigma_{x}\varsigma_{y} - 12\varsigma_{xx}\varsigma_{x}\varsigma_{x}\varsigma_{xy}$$

$$+ 4\varsigma_{xxx}\varsigma_{x}\varsigma_{z} - 12\varsigma_{xx}\varsigma_{x}\varsigma_{xz} - 2\varsigma\varsigma_{y}\varsigma_{xxxx} - 2\varsigma\varsigma_{xt}\varsigma_{y}$$

$$- 2\varsigma\varsigma_{yt}\varsigma_{x} - 2\varsigma\varsigma_{xy}\varsigma_{t} - 2\varsigma\varsigma_{yz}\varsigma_{t} - 8\varsigma_{xxxy}\varsigma_{x}\varsigma$$

$$- 2\varsigma\varsigma_{tz}\varsigma_{x} - 2\varsigma\varsigma_{tx}\varsigma_{z} - 2\varsigma_{xt}\varsigma_{z}\varsigma + 4\varsigma\varsigma_{xxx}\varsigma_{xy} \qquad (3)$$

$$+ 12\varsigma_{xxy}\varsigma_{x}\varsigma_{x} - 2\varsigma\varsigma_{z}\varsigma_{xxxx} - 8\varsigma\varsigma_{x}\varsigma_{xxxz} + 4\varsigma\varsigma_{xxx}\varsigma_{xz}$$

$$+ 12\varsigma_{xxz}\varsigma_{x}\varsigma_{x} + 2\varsigma_{txy}\varsigma\varsigma + 2\varsigma\varsigma\varsigma_{txz} + 2\varsigma\varsigma\varsigma_{yxxxx}$$

$$+ 2\varsigma\varsigma\varsigma_{zxxxx} = 0$$

The change expands the nonlinearity; however, it approves us to expect the test work. In [13], Zhang utilized the Hirota bilinearity with Bell polynomials hypotheses to create some lump soliton, lump kink arrangements, and irregularity with one-stripe soliton and with two-stripe lump solitons for (1).

2. Lump Soliton Solutions for BLMP Equation

To create lump arrangement, we deem that

$$\varsigma = g^2 + h^2 + \varepsilon_{11},$$

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$$g = \varepsilon_1 x + \varepsilon_2 y + \varepsilon_3 t + \varepsilon_4 z + \varepsilon_5,$$

$$h = \varepsilon_6 x + \varepsilon_7 y + \varepsilon_8 t + \varepsilon_9 z + \varepsilon_{10}.$$
(4)

where ε_i , $i = 1 \dots 11$, are genuine obscure that will be resolved consequently. By direct substitution from (4) in (3) and gathering the coefficients of polynomials in x, y, t, and z, we acquire a nonlinear algebraic system in ε_i ; by solving those equations with aid of Maple, we get some sets of solutions as follows:

$$\varepsilon_{1} = \varepsilon_{1},$$

$$\varepsilon_{2} = -\frac{\varepsilon_{1}^{2}\varepsilon_{9} + \varepsilon_{6}^{2}\varepsilon_{9} + 2\varepsilon_{6}^{2}\varepsilon_{7}}{2\varepsilon_{1}\varepsilon_{6}},$$

$$\varepsilon_{3} = 0,$$

$$\varepsilon_{4} = \frac{\varepsilon_{9}\left(\varepsilon_{1}^{2} - \varepsilon_{6}^{2}\right)}{2\varepsilon_{1}\varepsilon_{6}},$$

$$\varepsilon_{5} = 0,$$
(5)
$$\varepsilon_{6} = \varepsilon_{6},$$

$$\varepsilon_{7} = \varepsilon_{7},$$

$$\varepsilon_{8} = 0,$$

$$\varepsilon_{9} = \varepsilon_{9},$$

$$\varepsilon_{10} = 0,$$

$$\varepsilon_{11} = \varepsilon_{11}$$

So,

$$\varsigma = \left(\varepsilon_1 x - \frac{\varepsilon_1^2 \varepsilon_9 + \varepsilon_6^2 \varepsilon_9 + 2\varepsilon_6^2 \varepsilon_7}{2\varepsilon_1 \varepsilon_6} y + \frac{\varepsilon_9 \left(\varepsilon_1^2 - \varepsilon_6^2\right)}{2\varepsilon_1 \varepsilon_6} z\right)^2$$
(6)
+ $\left(\varepsilon_6 x + \varepsilon_7 y + \varepsilon_9 z\right)^2 + \varepsilon_{11}$

Utilizing (2), the solution of (1) has the form

$$v = -\frac{2\left(2\left(\varepsilon_{1}x - \left(\left(\varepsilon_{1}^{2}\varepsilon_{9} + \varepsilon_{6}^{2}\varepsilon_{9} + 2\varepsilon_{6}^{2}\varepsilon_{7}\right)/2\varepsilon_{1}\varepsilon_{6}\right)y + \left(\varepsilon_{9}\left(\varepsilon_{1}^{2} - \varepsilon_{6}^{2}\right)/2\varepsilon_{1}\varepsilon_{6}\right)z\right)\varepsilon_{1} + 2\left(\varepsilon_{6}x + \varepsilon_{7}y + \varepsilon_{9}z\right)\varepsilon_{6}\right)}{\varsigma}$$
(7)

Substituting (5) and (6) in (7) forms lump-kink solution as shown in Figure 1 with $\varepsilon_1 = \varepsilon_9 = \varepsilon_6 = 2$, $\varepsilon_7 = 1$, and $\varepsilon_8 = 1$.

3. Interaction Solutions

3.1. Lump Soliton with One-Stripe Wave. Assume that the test work is a confederation of quadratic function with exponential function as follows:

$$\varsigma = g^2 + h^2 + \varepsilon_{11} + e^{\xi_1 x + \xi_2 y + \xi_3 t + \xi_4 z + \xi_5},$$

$$g = \varepsilon_1 x + \varepsilon_2 y + \varepsilon_3 t + \varepsilon_4 z + \varepsilon_5,$$

$$h = \varepsilon_6 x + \varepsilon_7 y + \varepsilon_8 t + \varepsilon_9 z + \varepsilon_{10}.$$
(8)

where ε_i , i = 1...11 and ξ_j , j = 1..5, are r genuine obscure constants that will be resolved later on. Utilizing the ansatz in (2),

$$v = 2 \frac{2\varepsilon_1 g + 2\varepsilon_6 h + k_1 e^{\xi_1 x + \xi_2 y + \xi_3 t + \xi_4 z + \xi_5}}{\varsigma}$$
(9)



FIGURE 1: Top: 3D plots for (7) at (a) z = 0, (b) at z = 5, and (c) at z = 10. Nethermost: contour plots for (7) at (d) z = 0, (e) at z = 5, and (f) at z = 10.

Complicated algebraic system is driven by substituting from (8) in (3) and collecting the coefficients of polynomials in x, y, t, and z. We solve the constructed system utilizing Maple and snaffle the following assortment of solutions:

the following assortment of solu-

$$\varepsilon_1 = 0,$$
 $\varepsilon_2 = -\varepsilon_4,$
 $\varepsilon_2 = -\varepsilon_4,$
 $\varepsilon_3 = \varepsilon_3,$
 $\varepsilon_4 = \varepsilon_4,$
 $\varepsilon_5 = \varepsilon_5,$
 $\varepsilon_6 = 0,$
 $\varepsilon_5 = \varepsilon_5,$
 $\varepsilon_6 = 0,$
 $\varepsilon_7 = -\varepsilon_9,$
 $\varepsilon_8 = \varepsilon_8,$
 $\varepsilon_1 = \varepsilon_{10},$
 $\varepsilon_1 = -\frac{\varepsilon_3\varepsilon_5\varepsilon_{10}^2}{\varepsilon_5\varepsilon_3 + \varepsilon_8\varepsilon_{10}},$
 $\varepsilon_1 = \varepsilon_{11},$
 $\varepsilon_1 = -\frac{\varepsilon_3\varepsilon_5\varepsilon_{10}^2}{\varepsilon_5\varepsilon_3 + \varepsilon_8\varepsilon_{10}},$
 $\varepsilon_1 = \varepsilon_1,$
 $\varepsilon_1 = -\frac{\varepsilon_3\varepsilon_5\varepsilon_{10}^2}{\varepsilon_5\varepsilon_3 + \varepsilon_8\varepsilon_{10}},$
 $\varepsilon_1 = \varepsilon_1,$
 $\varepsilon_1 = -\frac{\varepsilon_1\varepsilon_5\varepsilon_{10}^2}{\varepsilon_5\varepsilon_3 + \varepsilon_8\varepsilon_{10}},$
 $\varepsilon_1 = \varepsilon_1,$
 $\varepsilon_2 = \varepsilon_1,$
 $\varepsilon_1 = \varepsilon_1,$
 $\varepsilon_2 = \varepsilon_1,$
 $\varepsilon_1 = \varepsilon_1,$
 $\varepsilon_2 = \varepsilon_2,$
 $\varepsilon_2 = \varepsilon_1,$
 $\varepsilon_2 = \varepsilon_1,$
 $\varepsilon_2 = \varepsilon_1,$
 $\varepsilon_2 = \varepsilon_2,$
 $\varepsilon_2 = \varepsilon$

 $\xi_2 = -\xi_4,$

 $\xi_3 = -\xi_1^3,$



FIGURE 2: Upper plots: 3D plots for (9) at (a) y = 0, at (b) y = 2, and at (c) y = 5 and z = 0. Contour plots (nethermost plots) for (9) at (d) y = 0, at (e) y = 2, and at (f) y = 5 and z = 0.

3.2. Interaction of Lump Solution with Rough Wave (Two-Stripe Solitons). We presume that the new ansatz is a collection of quadratic function and hyperbolic function as follows:

$$\begin{split} \varsigma &= g^2 + h^2 + \varepsilon_{11} \\ &+ \cosh \left(\xi_1 x + \xi_2 y + \xi_3 t + \xi_4 z + \xi_5 \right), \end{split}$$

$$g = \varepsilon_1 x + \varepsilon_2 y + \varepsilon_3 t + \varepsilon_4 z + \varepsilon_5,$$

$$h = \varepsilon_6 x + \varepsilon_7 y + \varepsilon_8 t + \varepsilon_9 z + \varepsilon_{10}.$$
(13)

Switching from (13) in (2), we embezzle a variety arrangement of answers for (1):

 $\varepsilon_7 = \varepsilon_7,$

 $\varepsilon_8 = 0$,

 $\varepsilon_9 = \varepsilon_9,$

 $\varepsilon_{10} = \varepsilon_{10},$

 $\varepsilon_{11} = \varepsilon_{11},$

$$v = \frac{2\left(2\left(\varepsilon_1 x - \varepsilon_4 y + \varepsilon_4 z\right)\varepsilon_1 + \sinh\left(\xi_1 x - \xi_4 y - \xi_1^3 t + \xi_4 z + \xi_5\right)\xi_1\right)}{\varsigma}$$
(14)

More intense computations were finished utilizing Maple software to get the obscure constants in accordance with representation form (13) in (3) and emulate the coefficients of x, y, t, and z to zero. Settling the subsequent nonlinear framework produces a few instances of the compelled parameters. For each situation, we back substitute in (13) as follows:

$\varepsilon_1 = \varepsilon_1,$	-11 -11,
	$\xi_1 = \xi_1,$
$\varepsilon_2 = -\varepsilon_4,$	$\xi_2 = -\xi_4,$
$\varepsilon_3 = 0,$	$k_3 = -\xi_1^3$,
$\varepsilon_4 = \varepsilon_4,$	$\xi_4 = \xi_4,$
$\varepsilon_5 = 0,$	$\xi_5 = \xi_5$
$\varepsilon_6 = 0,$	

(15)



FIGURE 3: Upper plots: 3D plots for (14) at (a) t = 0, at (b) t = 1, and at (c) t = 2 and z = 0. Contour plots (nethermost plots) for (14) at (d) t = 0, at (e) t = 1, and at (f) t = 2 and z = 0.

Substituting from (15) into (13),

$$\varsigma = (\varepsilon_1 x - \varepsilon_4 y + \varepsilon_4 z)^2 + (\varepsilon_7 y + \varepsilon_9 z + \varepsilon_{10})^2 + \varepsilon_{11} + \cosh(\xi_1 x - \xi_4 y - \xi_1^3 t + \xi_4 z + \xi_5)$$
(16)

Through a similar system, we get the arrangements of (1) and plot in Figure 3 for $\varepsilon_2 = -1$, $\varepsilon_6 = 1$, $\varepsilon_7 = 2$, $\varepsilon_5 = 1$, $\varepsilon_{10} = 0$, $\varepsilon_8 = 1$, $\varepsilon_{11} = 1$, $\xi_1 = 1$, $\xi_2 = 1$, $\xi_5 = 1$.

4. Conclusions

In this work, we constructed lump solutions and mixed solution involving lump waves and solitons for the incompressible fluid system (1) via bilinear method and symbolic computation. Starting form Cole-Hopf transformation which is that investigated by Singular Manifold method with twoterm truncated series, we drive some of the new and novel lump-solitons: lump-kink, lump interacted with one-stripe soliton or kink and interacted lump with two-stripe soliton, or kink wave after many complicated calculations utilizing Maple software. The three-dimensional plots and contour plots are presented for all solutions that we obtained. Via our intensive search, there is no one to investigate these types of solutions for (1). Our work is important for an interaction between lumps and to better get these frameworks.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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